A THREE-DIMENSIONAL FINITE-DIFFERENCE CALCULATION OF EQUIVALENT CAPACITANCES OF COPLANAR WAVEGUIDE DISCONTINUITIES

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ABSTRACT

A finite difference method is applied to three-dimensional multilayered shielded structures containing planar waveguide discontinuities. The electric field distribution inside the shielded structure is computed and the equivalent capacitances of the discontinuities are determined. Open-ends and gaps in various coplanar waveguides with one and two layer substrates as well as more complicated structures like interdigitated capacitors are examined. The calculated results are in good agreement with accurate measurements and results from other methods.

INTRODUCTION

As is well known, line-discontinuities can approximatly be described by lumped element equivalent circuits, as long as their geometrical dimensions are small compared to the wavelength so that the phase variation across the discontinuity can be neglected. In case of coplanar waveguides and because of the small dependence of the component dimensions on substrate thickness, the geometrical size of the components can be chosen small and therefore the condition mentioned above is fulfiled. This justifies the assumption that the frequency, beyond which the quasi-static results of coplanar waveguide discontinuities are no longer valid, may be higher than for comparable microstrip structures.

In order to test the validity of quasi-static calculations, the equivalent circuit parameters of some discontinuities in coplanar waveguides are determined using the electrostatic finite difference method. Well known circuit models are then used to obtain the circuit components from scattering parameters measured up to 25 GHz using an optimization routine. The camparison between calculated and measured parameters are used to indicate the accuracy of the calculation.

FINITE DIFFERENCE FORMULATION

Analogous to the finite difference method described in [1]-[3], a three- dimensional finite difference expression is evaluated for the calculation of the electrical potential distribution inside a shielded (electric or magnetic walls) structure on multilayered substrate material (Fig. 1). A magnetic wall is assumed, where a transmission line intersects the shielding. The bounded region is divided into elementary boxes using a three-dimensional non equidistant cartesian grid.

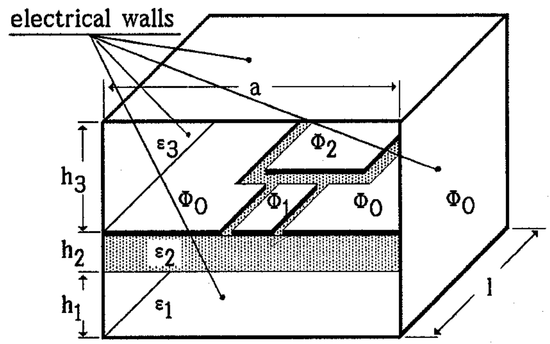
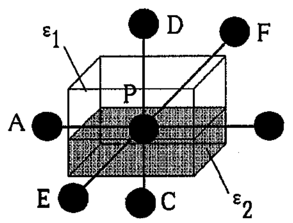


Figure 1: Three-dimensional multilayered shielded structure containing a coplanar discontinuity.

Laplace's equation is used for the finite difference formulation of the electrical potential at any grid-point P as follows:



$$\left(\frac{\varepsilon_1 d+\varepsilon_2 c}{ab}+\frac{\varepsilon_1}{d}+\frac{\varepsilon_2}{c}+\frac{\varepsilon_1 d+\varepsilon_2 c}{ef}\right)\Phi_P = \frac{\varepsilon_1 d+\varepsilon_2 c}{(a+b)}\left(\frac{\Phi_A}{a}+\frac{\Phi_B}{b}\right) + \frac{\varepsilon_2}{c}\Phi_C+\frac{\varepsilon_1}{d}\Phi_D+\frac{\varepsilon_1 d+\varepsilon_2 c}{(e+f)}\left(\frac{\Phi_E}{e}+\frac{\Phi_F}{f}\right) . \qquad (1)$$

For the solution of the resulting equation system, the "successive relaxation" method is used. In this method the potential distribution inside the shielding can be determined starting with assumed potential values at all the grid-points and modifying these as follows:

$$\Phi_{new} = \Phi_{old} - k \cdot R, \tag{2}$$

where R is the difference between Φ_{old} and the value given by equation (1), and $1<k<2$ is a constant which determines the speed of convergence. The electrical field at any grid-point can be calculated as:

$$\vec{E} = -\vec{\nabla}\Phi . \tag{3}$$

EQUIVALENT CAPACITANCES

For the following description, the configuration shown in Fig. 2 is considered. The field distribution calculated by equation (3) is used to find both the total charge on the inner conductor (Q_{total}) and the charge per unit length on the connected transmission line (Q').

$$Q' = \varepsilon_0 \varepsilon_r \oint_C E_n \cdot ds , \tag{4}$$

$$Q_{total} = \varepsilon_0 \varepsilon_r \int_A E_n \cdot dA . \tag{5}$$

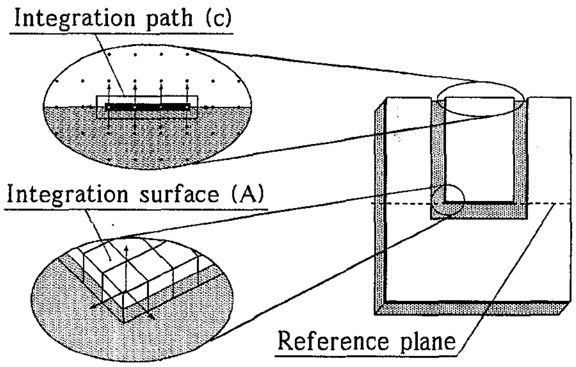


Figure 2: Evaluation of the equivalent capacitance of a coplanar waveguide open-end.

If the potential difference between the inner conductor and the ground planes is V, the capacitance associated to the discontinuity can be calculated from the difference between the total charge Q_{total} and the charge per unit length of the connected transmission line.

$$C_{eq} = V \cdot (Q_{total} - l \cdot Q'), \tag{6}$$

where l is defined as the distance between the reference plane of the discontinuity and the magnetic wall.

TESTING STRUCTURES

1. Open-Ends

In a first approximation, the abrupt open-end of a transmission line can be described by an equivalent capacitance (Fig. 3b). Another usable model is an extended length Δl of the transmission line which is defined as the ratio of equivalent capacitance to capacitance per unit length of the line (Fig. 3c).

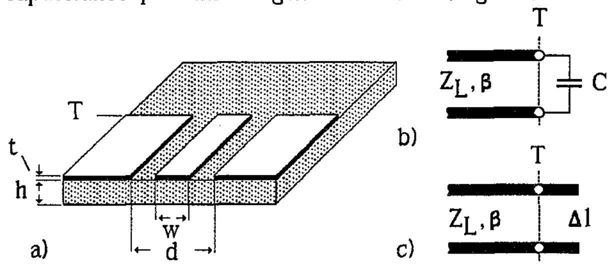


Figure 3: Open-ended coplanar waveguide and its equivalent circuit models.

To test the accuracy of the chosen method, the capacitances of microstrip open-ends on various substrate materials are plotted in Fig. 4 as a function of width to substrate height ratio together with the results presented by [4]. As shown in this figure, the data calculated with the presented method are in good agreement with accurate values based on fullwave spectral domain calculations [4].

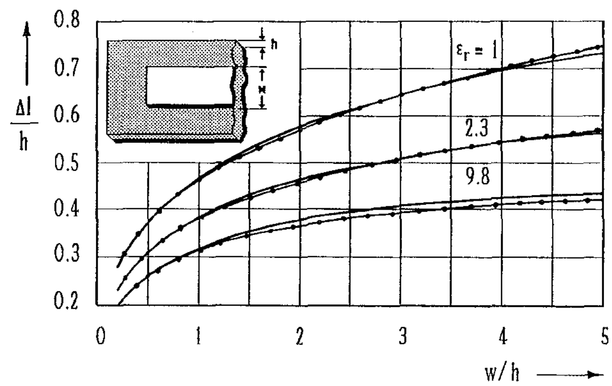


Figure 4. Equivalent length Δl of an open-ended microstrip line (solid lines) compared to fullwave spectral domain results at 4 GHz (doted lines).

The next testing structure is the coplanar waveguide open-end shown in Fig. 3a. The calculated data are plotted in Fig. 5 against the width to spacing ratio for different substrate thicknesses together with measured values. As expected, the equivalent capacitance increases steadily with the width of the inner conductor.

In some cases the ground planes of the coplanar waveguide are connected close to the open end of the line. The equivalent parameters of such a structure are plotted in Fig. 6. It may be noted that for values g>d the variation of the equivalent capacitance is negligible.

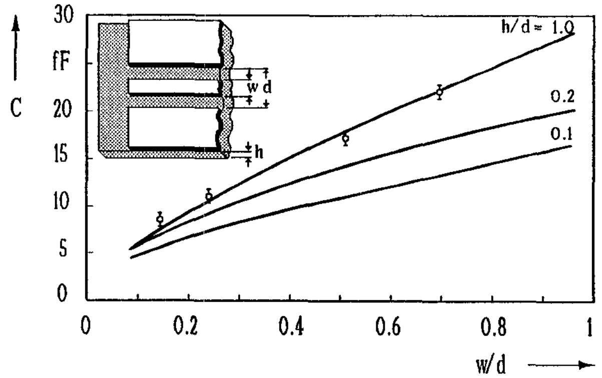


Figure 5: Calculated equivalent capacitance of an open-ended coplanar waveguide (ε_r=9.8, h=0.635 mm) together with measured results for h/d=1.0

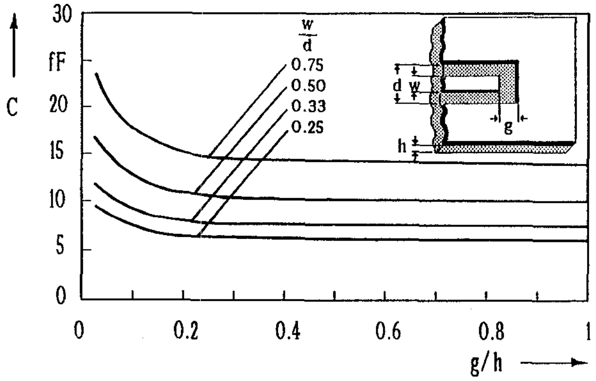
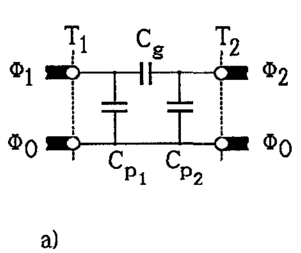


Figure 6: Equivalent capacitance of an open-ended coplanar waveguide with connected ground planes (ε_r=12.9, h=0.635 mm, d/h=0.6).

2. Gaps

The two-port network model shown in Fig. 7a is used for the characterization of the gap discontinuity. The equivalent capacitances C_g, C_{p1} and C_{p2} are calculated using two potential-configurations (even and odd mode) as shown in Fig. 7b.



even mode

$$\left.\begin{array}{r}\Phi_1=\Phi_2=1\,V\\ \Phi 0=0\,V\end{array}\right\} \Rightarrow C_{P_1}, C_{P_2}$$

odd mode

$$\left.\begin{array}{r}\Phi_1=-\Phi_2=1\,V\\ \Phi 0=0\,V\end{array}\right\} \Rightarrow C_g$$

b)

Figure 7: Pi-network model of a coplanar line gap

Several symmetric and asymmetric gaps in coplanar waveguides have been investigated. The results for symmetric gaps between two coplanar waveguides are presented in Fig. 8 together with measured values.

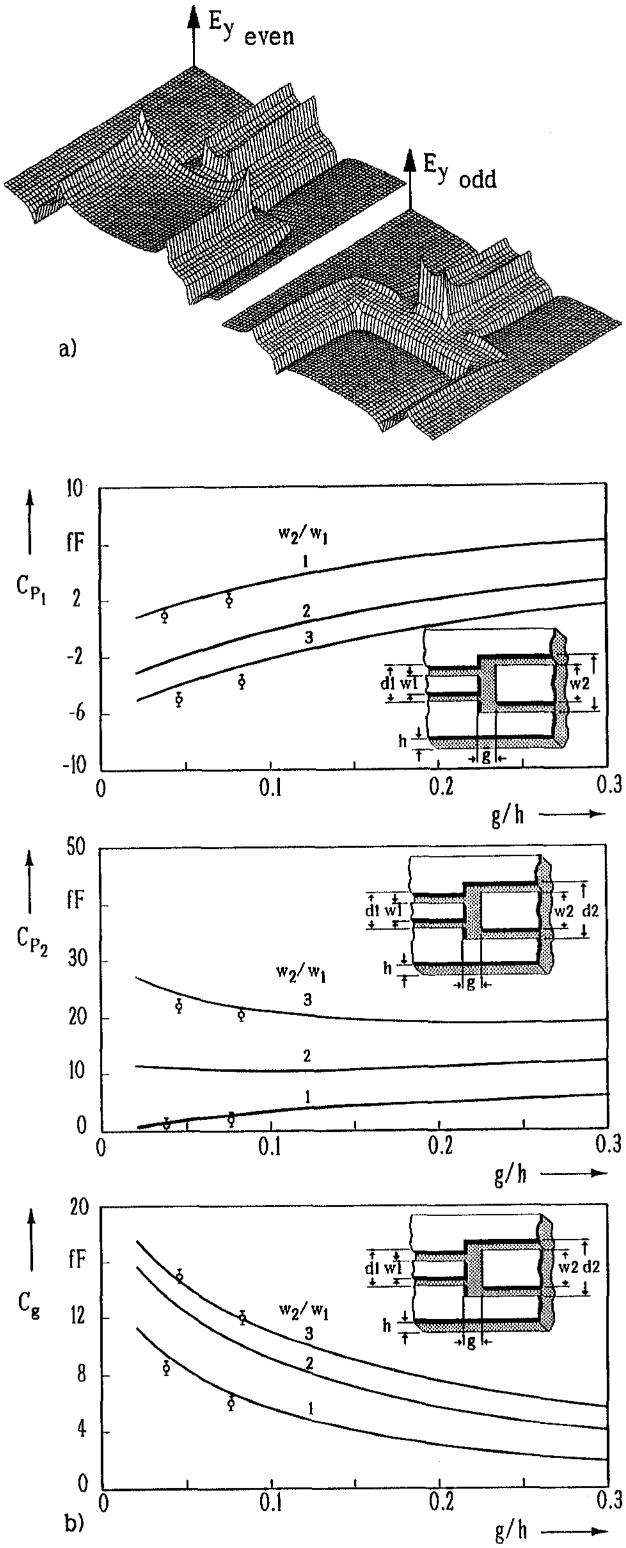


Figure 8: a) y-component of transversal electric field for the even- and odd-mode. b) Equivalent capacitances of a gap in a coplanar waveguide (ε_r=9.8, h=0.635 mm, w_1/h=0.2, w_1/d_1=w_2/d_2=0.56).

3. Interdigitated Capacitors

As mentioned previously, the method presented here can also be applied on more complicated structures. This is because of the fact that the equivalent parameters are calculated from the potential distribution which can be determined independently of the conductor configuration inside the shielding. As an example, the equivalent capacitances of coplanar interdigitated capacitors are calculated based on the two-port model of Fig. 7a. The comparison of calculated data with measured scattering parameters in Fig. 9 shows that in spite of the simplifed circuit model the agreement is very well up to 25 GHz. As shown in Table 1, the agreement is also good (considering the small values of the capacitances of several femto-farads) for other dimensions and substrate materials.

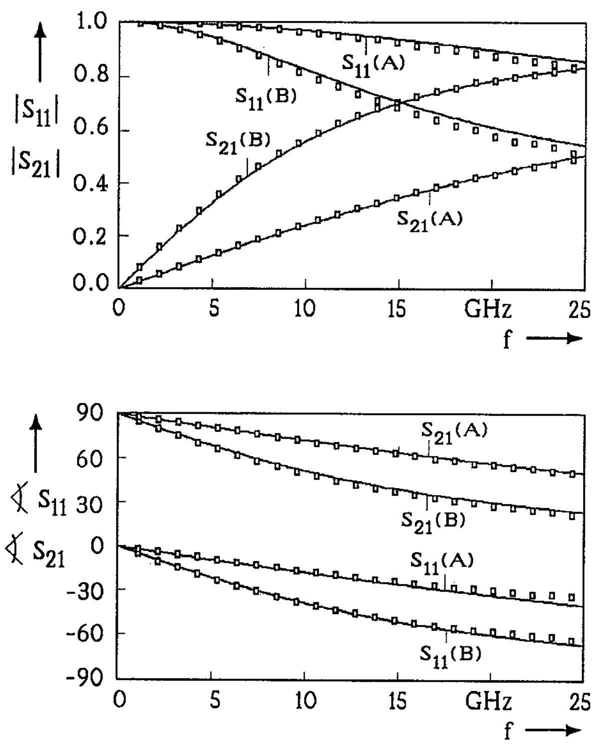
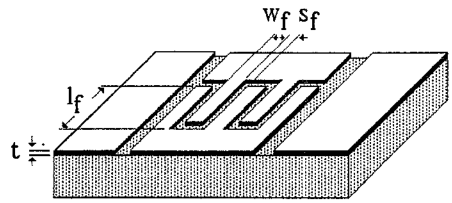


Figure 9: Calculated (——) and measured (▫ ▫ ▫) scattering parameters of coplanar interdigitated capacitors. (see Table 1 for dimensions)

The parallel capacitances C_{p1} and C_{p2} depend mainly on the length of the fingers while the coupling capacitance C_g is influenced by both the number and the length of fingers. Also the conductor metallization thickness has a large effect on C_g especially for broad fingers and small distances between the fingers. For very narrow and long fingers, the inductive effect can no longer be neglected especially for a small number of fingers.



	ε_r	n_f	w_f	s_f	l_f	t	C_{p1}	C_{p2}	C_g	
			μm	μm	μm	μm	fF	fF	fF	
cal.	12.9	4	17	3	100	3	9.65	9.65	40.1	A
meas.							11	11	41	
cal.	12.9	10	12	2	100	3	9.9	9.9	113.3	B
meas.							11	11	116	
cal.	12.9	4	17	3	200	3	19.2	19.2	73	
meas.							20	20	76	
cal.	9.8	5	38	25	200	5	22.94	10.78	55	
meas.							23	11	56	
cal.	9.8	7	38	25	200	5	21.78	11.64	85	
meas.							22	12	86	

Table 1. Equivalent parameters of coplanar interdigitated capacitors as compared to measured values.

CONCLUSION

The advantage of the method presented here is its flexibility regarding the number of substrate layers and the conductor configuration. It is therefore applicable to many planar waveguides supporting TEM-waves. The method can be easily extended to more complicated structures such as interdigital capacitors, waveguide transitions (microstrip to coplanar) or air-bridges. The effect of metallization thickness can also be investigated. The computing time and the storage requirement is considerably reduced by using a non-equidistant grid.

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